

Dirac-Fock Current and Magnetic-Moment in the Relativistic Approach

Tomoyuki Maruyama

Advanced Science Research Center,
Japan Atomic Energy Research Institute,
Tokai, Ibaraki 319-11, Japan

Abstract

We define the Dirac-Fock current to hold the current conservation in the momentum-dependent Dirac fields, and discuss its effect on the magnetic-moment in the nuclear matter. It is shown that the Dirac-Fock current can largely reduce the magnetic-moment even in the isovector case, whose value has been too big due to the small effective mass in the usual relativistic Hartree approximation.

The past decades have seen many successes in the relativistic treatment of the nuclear many-body problem. The relativistic framework has big advantages in several aspects [1]: a useful Dirac phenomenology for the description of nucleon-nucleus scattering [2], the natural incorporation of the spin-orbit force [1] and the saturation properties in the microscopic treatment with the Dirac Brueckner Hartree-Fock (DBHF) approach [3].

These results conclude that there are large attractive scalar and repulsive vector-fields, and that the nucleon effective mass is very small in the medium. However this small effective mass makes some troubles in the nuclear properties: too big magnetic-moment [4] and too big excitation energy of the isoscalar giant quadrupole resonance (ISGQR) state [5]. As for the isoscalar magnetic-moment, this enhancement is canceled by the Ring-Diagram contribution [6]; this relation is completely realized by the gauge invariance [7]. As for the isovector one, however, this contribution does not play a strong role because the symmetry force is not efficiently large.

In this subject, by the way, most of people believe that the momentum-dependence of the Dirac fields is negligible in the low energy region, particularly below the Fermi level. A momentum-dependence of the Schrödinger equivalent potential automatically emerges as a consequence of the Lorentz transformation properties of the vector-fields without any explicit momentum-dependence of the scalar and vector fields. In fact, only very small momentum-dependence has appeared in the relativistic Hartree-Fock calculation [8].

In the high energy region, however, the vector-fields must become very small to explain the optical potential of the proton-nucleus elastic scattering [2, 9], and the transverse flow in the heavy-ion collisions [10]. This fact suggests us that the momentum-dependent part is not actually small though it has not been clearly seen in the low energy phenomena. After this consideration we naturally have a question whether the momentum-dependence of the Dirac fields is really not important. Since the momentum-dependent fields break the current conservation, furthermore, we have to define the new current caused by the vertex correction.

In this paper, thus, we define a new current to hold the current conservation in the momentum-dependent Dirac fields, and discuss its effect on the magnetic-moment.

Let us consider the infinite nuclear matter system. The nucleon propagator in the self-energy Σ is given by

$$S^{-1}(p) = \gamma p - M - \Sigma(p), \quad (1)$$

where $\Sigma(p)$ has a Lorentz scalar part U_s and a Lorentz vector part $U_\mu(p)$ as

$$\Sigma(p) = -U_s(p) + \gamma^\mu U_\mu(p). \quad (2)$$

For the future convenience we define the effective mass and the kinetic momentum as

$$\begin{aligned} M^*(p) &= M - U_s(p), \\ \Pi_\mu(p) &= p - U_\mu(p). \end{aligned} \quad (3)$$

The single particle energy with momentum \mathbf{p} is obtained as

$$\begin{aligned} \varepsilon(\mathbf{p}) &= p_0|_{on-mass-shell} \\ &= \sqrt{\Pi^2(\mathbf{p}) + M^{*2}} + U_0(\mathbf{p}). \end{aligned} \quad (4)$$

Then the detailed form of the nucleon propagator eq. (1) is represented by

$$S(p) = \{\Pi(p) + M^*(p)\} \left\{ \frac{1}{\Pi^2 - M^{*2} + i\delta} - 2i\pi n(\mathbf{p})\delta(\Pi^2 - M^{*2}) \right\} \quad (5)$$

with the momentum distribution $n(\mathbf{p})$.

If the self-energy has a momentum-dependence, the current operator must be also changed. We then define the current vertex $\Gamma^\mu(p+q, p)$ as

$$\Gamma^\mu(p+q, p) = \gamma^\mu + \Lambda^\mu(p+q, p). \quad (6)$$

From the Ward-Takahashi identity the density-dependent vertex correction Λ^μ must be satisfied with

$$q_\mu \Lambda^\mu(p+q, p) = -\Sigma(p+q) + \Sigma(p). \quad (7)$$

We call the new additional current $\hat{j} = \bar{\psi}\Lambda^\mu\psi$ the Dirac-Fock current, which is produced by the momentum-dependence of the Dirac fields, because the momentum-dependent part is mainly caused by the Fock-diagram. Next we discuss a role of this current to the magnetic-moment.

In the limit $q \rightarrow 0$ this vertex correction becomes

$$\Lambda^\mu(p) = \lim_{q \rightarrow 0} \Lambda^\mu(p+q, p) = -\frac{\partial}{\partial p_\mu} \Sigma(p). \quad (8)$$

Using the above vertex correction, the current density of the whole system is given as

$$\begin{aligned} \mathbf{j}_\mu &= \int \frac{d^4p}{(2\pi)^4} \text{Tr}\{\Gamma_\mu S(p)\} \\ &= \int \frac{d^3p}{(2\pi)^3} n(\mathbf{p}) \frac{\tilde{\Pi}_\mu(p)}{\tilde{\Pi}_0(p)}, \end{aligned} \quad (9)$$

where $\tilde{\Pi}_\mu$ is defined by

$$\begin{aligned} \tilde{\Pi}_\mu(p) &= \frac{\partial}{\partial p^\mu} V(p) \\ &= \frac{1}{2} \frac{\partial}{\partial p^\mu} \{\Pi^2(p) - M^{*2}(p)\}. \end{aligned} \quad (10)$$

Let us consider the one-particle state on the Fermi surface. The nucleon space current can be written as

$$\mathbf{j} = \frac{\tilde{\Pi}(p)}{\tilde{\Pi}_0(p)}|_{|\mathbf{p}|=p_F} = \mathbf{D}\mathbf{p}\varepsilon(\mathbf{p})|_{|\mathbf{p}|=p_F}, \quad (11)$$

where the total derivative $\mathbf{D}\mathbf{p}$ is defined on the on-mass-shell condition: $p_0 = \varepsilon(\mathbf{p})$. The above equation completely agrees with that derived by the semi-classical way [9].

In the non-relativistic framework the effective mass is defined by

$$M_L^* = \left(2 \frac{d}{d\mathbf{p}^2} \varepsilon(\mathbf{p})\right)^{-1}|_{|\mathbf{p}|=p_F}, \quad (12)$$

which is so called the '**Landau mass**'. Then the above spacial current is

$$\mathbf{j} = \frac{\mathbf{p}_F}{M_L^*}. \quad (13)$$

In our case including the momentum-dependent Dirac fields, the value of the Landau mass M_L^* cannot be uniquely determined from the relativistic effective mass M^* while in the Hartree approximation the Landau mass becomes $M_L^* = \Pi_0(\mathbf{p}_F)$.

Now the main problem is how to determine the value of the Landau mass M_L^* . In the theoretical aspect it should be derived by making use of DBHF with the momentum-dependent self-energies. However it is not so easy technically. Recently Huber et al. [12] have introduced the momentum-dependence into the DBHF calculation, but they have not given this value. As discussed later, very small contribution of the momentum-dependence largely changes the value of M_L^* even if it does not change the saturation property. Hence the ambiguity involved in their calculation, for example, the basic nuclear force, is not as small as to fix this value.

Hence we need to determine the value of M_L^* from the experimental analysis. One candidate is the analysis of $p - A$ elastic scattering data, but this method cannot be used in very low energy region near the Fermi level. In other observables the excitation energy of ISGQR must be most sensitive to the Landau mass because this energy is almost determined by the mean kinetic energy, and the surface effect is not significant. In addition, the relativistic approach does not change its microscopic picture [5]; the strength of the resorting force is determined only by $d\varepsilon(\mathbf{p})/d\mathbf{p}^2|_{|\mathbf{p}|=p_F}$. T. Suzuki [11] has given $M_L^*/M \approx 0.85$ from this analysis. Using this value the enhancement of the space current from the free one is about 15 %. Namely the enhancement of the magnetic-moment from the Schmidt value is also about 15 %.

When we use the typical value of $M^*/M \approx 0.55 - 0.6$ [2, 3, 13], the enhancement is about 50 % in the Hartree approximation. The effect of the momentum-dependence largely improves the magnetic-moment.

Finally we discuss how much degree of momentum-dependence is necessary to get $M_L^*/M = 0.85$ with $M^*/M = 0.6$. For simplicity we neglect the space part of the vector fields, which is really small in the Hartree-Fock calculation. The single particle energy is written as

$$\varepsilon(\mathbf{p}) = \sqrt{\mathbf{p}^2 + M^{*2}(\mathbf{p})} + U_0(\mathbf{p}). \quad (14)$$

The Landau mass is given by

$$M_L^* = \Pi_0(p_F)(1 - M^*(p_F)\frac{dU_s}{d\mathbf{p}^2} + 2\Pi_0(p_F)\frac{dU_0}{d\mathbf{p}^2})^{-1}. \quad (15)$$

In the case that $dU_s/d\mathbf{p}^2 = 0$, for example,

$$\frac{U_0(p_F) - U_0(0)}{U_0(p_F)} \approx 2p_F^2 \frac{dU_0}{d\mathbf{p}^2} / U_0 \approx 0.04 \quad (16)$$

with $M_L^*/M = 0.85$ and $M^*/M = 0.6$. Only 4 % contribution of the momentum-dependence of the vector-field can reduce the magnetic-moment about 20 %. Hence this

argument does not change the saturation property and the nuclear equation of state as seen in Ref. [10].

We can easily imagine that the one-pion exchange force largely produce the momentum-dependence because the interaction range is largest. The Fock self-energy from it with the pseudo-scalar coupling gives $dU_s/d\mathbf{p}^2 > 0$ and $dU_0/d\mathbf{p}^2 < 0$, where both contribution can enlarge the Landau mass. Hence the scenario in this paper is very possible.

In summary we have defined a new current, so called Dirac-Fock current, which is consistent to the momentum-dependent Dirac fields, and have shown that the strength of the spacial current is determined by the Landau mass M_L^* independently of the effective mass M^* . When we take the value of M_L^* from the result of ISGQR, the enhancement of the magnetic-moment should be 15 % from the Schmidt value even in the isovector case. In this case the Dirac fields does not need to be varied largely.

In fact the observed magnetic-moment includes the anomalous part, which is not enhanced by the small effective mass, and is given by the nucleon on the nuclear surface where the effective mass is bigger than that at the saturated nuclear matter. Therefore the actual enhancement of the isovector magnetic-moment should be less than 10 %. This difference is permissible because the actual isovector magnetic-moment has other contributions such as the exchange current.

As seen in this paper the momentum-dependent parts, which are non-local in the finite nuclei, are very effective even if these parts are small. In future we need to discuss effects of the non-local parts of Dirac fields to study nuclear structure and reactions.

Since the Fock effects are largely seen in the wide energy region [9, 10], this Dirac-Fock current also plays an important role with the large momentum-transfer q . In future we need to discuss effects of this current in the high momentum transfer phenomena such as the quasielastic electron scattering [14]. For this discussion we will have to figure out the detailed form of this current which has not been given in this paper. In order to get final conclusions, of course, we must also take into account the Ring-Diagram [15] which contributes to the final results largely.

References

- [1] B.D. Serot and J. D. Walecka, The relativistic Nuclear Many Body Problem. In J. W. Negele and E. Vogt, editors, *Adv.Nucl.Phys.***Vol.16**, page 1, Plenum Press, 1986, and reference therein.
- [2] B. C. Clark, S. Hama, R. L. Mercer, L. Ray and B. D. Serot, *Phys. Rev. Lett.* **50** (1983) 1644;
J. A. Tjon and S. J. Wallace, *Phys. Rev.* **C36** (1987) 1085;
S. Hama, B. C. Clark, E. D. Cooper, H. S. Sherif and R. L. Mercer, *Phys. Rev.* **C41** (1990) 2737.
- [3] W. Botermans and R. Malfliet, *Phys. Rep.* **198** (1990) 115,
R. Brockmann and R. Machleidt, *Phys. Rev.* **C42** (1990) 1965.
- [4] A. Bouyssy, S. Marcos and J.F. Mathiot, *Nucl. Phys.***A415** (1984) 497.

- [5] S. Nishizaki, H. Kurasawa and T. Suzuki, Nucl. Phys. **A462** (1987) 687.
- [6] H. Kurasawa and T. Suzuki, Phys. Lett. **165B** (1985) 234.
- [7] W. Bentz, A. Arima, H. Hyuga, K. Shimizu and K. Yazaki, Nucl. Phys. **A436** (1985) 593.
- [8] C. J. Horowitz and B. D. Serot, Nucl. Phys. **A399** (1983) 529.
- [9] K. Weber, B. Blättel, W. Cassing, H.-C. Dönges, V. Koch, A. Lang and U. Mosel, Nucl. Phys. **A539** (1992) 713.
- [10] T. Maruyama, B. Blättel, W. Cassing, A. Lang, U. Mosel, K. Weber, Phys. Lett **B297** (1992) 228,
T. Maruyama, W. Cassing, U. Mosel, S. Teis and K. Weber, Nucl. Phys **A552** (1994) 571.
- [11] T. Suzuki, Prog. Theor. Phys. **64** (1980) 1627.
- [12] H. Huber, F. Weber and M. K. Weigel, Phys. Rev. **C51** (1995) 1790.
- [13] S. Qing-biao and F. Da-chun, Phys. Rev. **C43** (1991) 2773.
- [14] G. Do Dang and N. Van Giai, Phys. Rev. **C30** (1984) 731,
S. Nishizaki, T. Maruyama, H. Kurasawa and T. Suzuki, Nucl. Phys. **A485** (1988) 515.
- [15] H. Kurasawa and T. Suzuki, Nucl. Phys. **A445** (1985) 685.